EGR 537–001 **NUMERICAL ANALYSIS** **Fall 2017**

J. M. McDonough
Departments of Mechanical Engineering and Mathematics
email: jmmcd@uky.edu

Numerical analysis is that branch of applied mathematics devoted to development of techniques to be employed in computation—*producing numbers.* In the present era of high-speed, relatively inexpensive digital computers this is an essential topic for engineers and scientists who must solve problems associated with predicting behavior of a broad spectrum of physical systems. While, in principle, one might expect to accomplish this via the usual software packages such as MatLab, it is crucial to understand how these packages work—what is actually happening inside them—so that if results are not as expected it is possible to understand the source of difficulty, and possibly fix it.

The goal of Section 001 of this course is to provide engineering (including computer science), math and physics students with a broad overview of basic numerical analysis organized in such a way as to naturally culminate, toward the end of the semester, in an introduction to the numerical solution of partial differential equations, thus leading to the ability to solve a large percentage of problems likely to be encountered in research and practice. Despite this broad coverage of topics, no previous mathematics background beyond sophomore calculus is presumed; but courses in analysis and/or differential equations are obviously helpful. Each topic will be introduced with a brief treatment of the underlying "pure" mathematics followed by widely-used appropriate numerical algorithms.

COURSE OUTLINE

I. Numerical Linear Algebra

- A. Solution of linear systems
  1. direct methods—Gaussian elimination, tridiagonal LU decomposition
  2. iterative methods—fixed-point iteration, Jacobi, Gauss–Seidel, SOR

B. The algebraic eigenvalue problem—power method

II. Solution of Non-linear Equations

A. Fixed-point iteration, revisited
B. Newton's method
C. Newton's method for systems

III. Approximation Theory

A. Approximation of functions
   1. non-linear least squares—an application of Newton's method
   2. polynomial interpolation
B. Numerical quadrature—approximation of definite integrals
   1. trapezoidal integration
   2. Richardson extrapolation
   3. Simpson's rule
C. Finite-difference approximation of derivatives
   1. forward, backward, centered approximation of first derivatives
   2. higher-order derivative approximation
D. Richardson extrapolation—in general
E. Grid-function convergence

IV. Ordinary Differential Equations (ODEs)

A. Initial-value problems
   1. some basic mathematical observations
   2. forward- and backward-Euler methods
   3. trapezoidal integration
   4. explicit trapezoidal—a 2^{nd}-order Runge–Kutta method
   5. stiff equations
B. Boundary-value problems
   1. mathematical formulation
   2. finite-difference approximation
   3. treatment of boundary conditions
   4. solution of the linear algebra problem
   5. treatment of coordinate singularities

6. quasilinearization of non-linear problems—a function-space Newton's method
7. introduction to the Galerkin procedure

V. Partial Differential Equations (PDEs)

A. Overview of solution methods

B. Classification of PDEs; well posedness of PDE problems

C. The heat equation
   1. basic mathematics
   2. forward-Euler/centered approximation—stability considerations
   3. the Crank–Nicolson method

D. Laplace's equation
   1. problem formulation and discretization
   2. solution via successive overrelaxation (SOR)

E. The wave equation
   1. mathematics of hyperbolic equations
   2. basic centered-difference approximation

HOMEWORK

There will be **three** homework assignments (each consisting of several separate problems) throughout the semester, with due dates (somewhat) negotiable; but all students must submit solutions on the same date, and this date will precede the due date of any subsequent assignment. **All problems will involve running high-level language (Fortran, Basic, C or C^{++}, MatLab) computer codes**. Codes for all but the simplest algorithms will be provided, but students are encouraged to write their own codes if they are proficient in using one of the noted higher-level languages.

Only one problem (to be selected randomly by me after the assignment has been turned in) from each set will be graded in detail, and the score from that problem will count 1/2 of the total score for the whole assignment. The remaining problems will be given equal weight to account for the other 1/2, and will be graded on a rather perfunctory basis. Homework sets will not necessarily be equally weighted, but an indication of relative weighting for each set will be provided.

EXAMINATIONS

There will be one (1) midterm examination given at approximately the middle of the semester (date, and other details, to be determined later). The two-hour final exam **will be comprehensive**, but will emphasize material from the second half of the semester; both exams will be closed book, closed notes, and will not require use of calculators—hence, no calculators or any other electronic devices will be permitted.

GRADING

Grades will be assigned with appropriate "curving" as needed, but anyone accumulating 90% or more of the total points for the course is guaranteed a grade of **A**. Total course point assignment will be based on the following distribution:

Homework (total)	30%
Midterm	30%
Final Exam	40%

RECOMMENDED READING

E. Isaacson and H. Keller, *Analysis of Numerical Methods*, Dover Pub. Co. (paperback)

J. M. McDonough, *Lectures in Basic Computational Numerical Analysis*, to be made available in PDF format from the UK Engineering website:

http://courses.engr.uky.edu/

OFFICE HOURS

By appointment—send email, or whenever you can catch me.