CFD of INCOMPRESSIBLE FLOW

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Computational fluid dynamics (CFD) is rapidly becoming one of the most important of engineering/physics disciplines. With the ever-increasing capability of supercomputers it is now possible to produce highly-accurate solutions to the full Navier–Stokes (N.–S.) equations in many physical circumstances. This implies that CFD will soon begin to serve both as a tool for engineering design and analysis and as a research instrument capable of elucidating new physical phenomena. Moreover, because essentially all life forms, as we know them on Earth, depend strongly on fluid motions for their existence and functioning, students from the biological sciences could also benefit from courses in CFD if they first acquire the necessary mathematical foundation. It is especially essential for graduate students in the thermal-fluid sciences to develop a reasonably good level of proficiency in this important area during their studies to be able to conduct first-rate research required for their graduate degrees.

The present course is intended to provide an introductory treatment of both the underlying mathematics of the incompressible N.–S. equations and the algorithms needed to solve them. It is important to recognize that success of an algorithm is strongly influenced by the degree to which it is able to address, and/or conform to, the underlying mathematics of solutions to these equations. Hence, in this course we place a fair amount of emphasis on the appropriateness of any given approach within the context of the regularity of solutions to the N.–S. equations in a given physical situation. But we also consider the more standard CFD topics such as treatment of the "cell Reynolds number problem," and somewhat related effects of aliasing of solutions, methods for maintaining the divergence-free condition on the velocity field of an incompressible N.–S. flow and various possible gridding alternatives related to this. We then present a basically chronologically-ordered treatment of the main incompressible N.–S. solution algorithms in use today.

Course Outline

I. Mathematics of the Navier–Stokes equations

   A. Why we should care about the mathematics
   B. Some function spaces
   C. Classical, weak and strong solutions
   D. Mathematical representations of the N.–S. equations
   E. Main theorems on existence, uniqueness and regularity of N.–S. solutions

II. Special Difficulties Arising in Numerical Solution of N.–S. Equations

   A. The problem of pressure-velocity coupling

      1. various forms of N.–S. equations used for numerical computation; their problems associated with well posedness
      2. alternative gridding arrangements—the problem of "checkerboard" solutions, and how to solve it

   B. Cell-*Re* and high-wavenumber aliasing problems

      1. some standard cell-*Re* treatments
      2. solution filtering

III. The Main Computational Algorithms

   A. Marker-and-cell (MAC) method

   B. SOLA-VOF

   C. Artificial compressibility

   D. Projection methods

   E. SIMPLE and variants; PISO

Course Requirements

There will be two homework exercises of approximately equal weight with regard to the final course grade. These will basically consist of writing/using computer codes to implement N.–S. equation solution algorithms for problems posed in Cartesian coordinates. This will account for 60% of the course grade. In addition, there will be an in-class, closed-book, comprehensive final examination given at the time published in the schedule of classes. This will count for 40% of the total course grade.

Lecture notes for the course will be available at the engineering course website,

http://courses.engr.uky.edu

There is no specific textbook for the course, but it is hoped that students will consult various of the references cited in the notes.